

# Análise de Circuitos em Corrente Alternada

## Resolução dos Exercícios Propostos

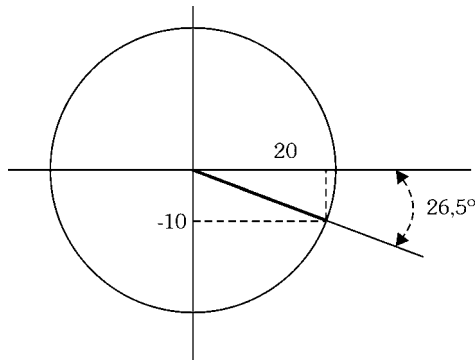
### Capítulo 1

1.1 - Converter na forma polar

a)  $z_1 = 20 - j10 \Rightarrow \operatorname{tg} \phi = 10/20 = 0,5 \Rightarrow \phi = -26,5^\circ$  (4º quadrante)

$$Z_1 = \sqrt{20^2 + 10^2} = 22,3$$

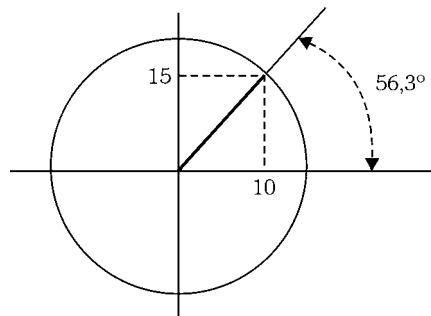
$$Z_1 = 22,3 \angle -26,5^\circ$$



b)  $z_2 = 10 + j15 \Rightarrow \operatorname{tg} \phi = 15/10 = 1,5 \Rightarrow \phi = 56,3^\circ$  (1º quadrante)

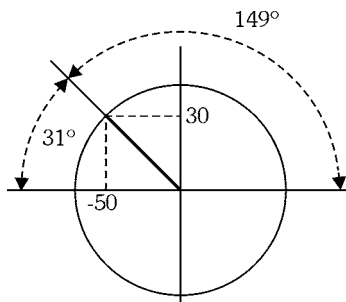
$$Z_2 = \sqrt{15^2 + 10^2} = 18$$

$$Z_2 = 18 \angle 56,3^\circ$$



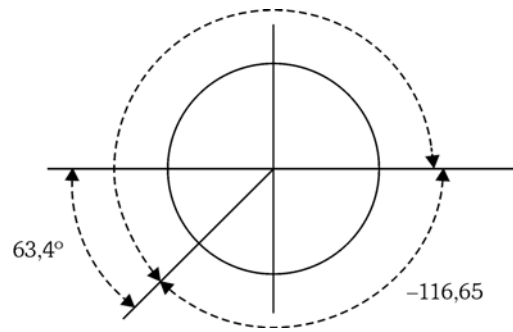
c)  $z_3 = -50 + j30 \Rightarrow \operatorname{tg} \phi = 30/50 = 0,6 \Rightarrow \phi = 31^\circ$  (2º Quadrante)

$$Z_3 = \sqrt{50^2 + 30^2} = 58,3 \quad Z_3 = 58 \angle 149^\circ$$

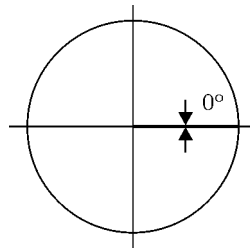


d)  $z_4 = -6 - j12 \Rightarrow \operatorname{tg} \phi = 12/6 = 2 \quad \phi = 63,4^\circ$

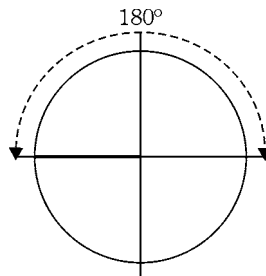
$Z_4 = \sqrt{(-6)^2 + (-12)^2} = 13,4 \quad Z_4 = 13,4 \angle 243,4^\circ = 13,4 \angle -116,5^\circ$



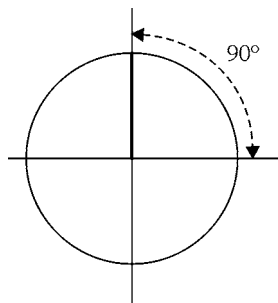
e)  $Z_5 = 5 \Rightarrow \phi = 0^\circ \quad Z_5 = 5 \angle 0^\circ$



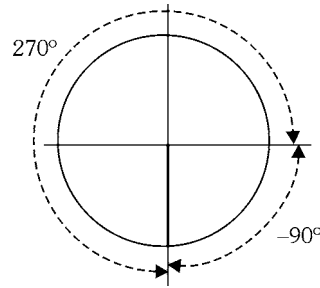
f)  $Z_6 = -15 \quad \phi = 180^\circ \quad Z_6 = 15 \angle 180^\circ$



g)  $Z_7 = j25 \Rightarrow \phi = 90^\circ \quad Z_7 = 25 \angle 90^\circ$

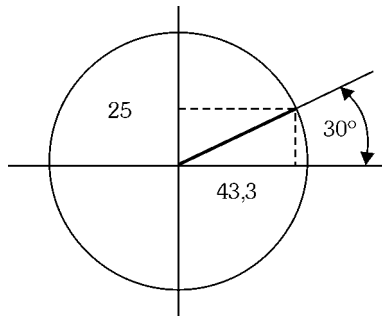


h)  $Z_8 = -j9$   $\phi = 270^\circ$  ou  $\phi = -90^\circ$   $Z_8 = 9 \angle 270^\circ = 9 \angle -90^\circ$

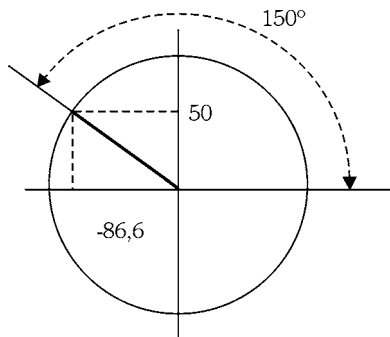


1.2 - Converter na forma cartesiana

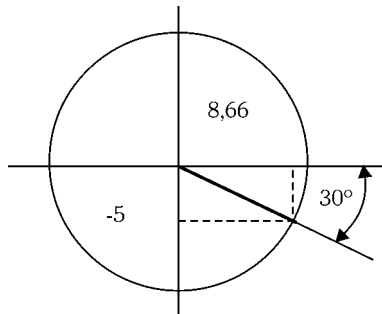
a)  $Z_1 = 50 \angle 30^\circ$   $Z_1 = 50 \cdot \cos 30 + j50 \cdot \sin 30 = 43,3 + j25$



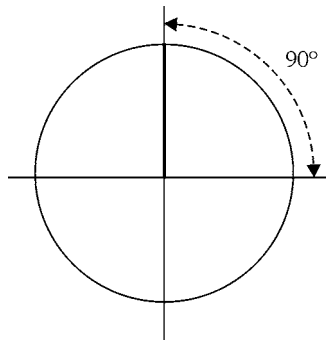
b)  $Z_2 = 100 \angle 150^\circ = 100 \cdot \cos 150 + j100 \cdot \sin 150 = -86,66 + j50$



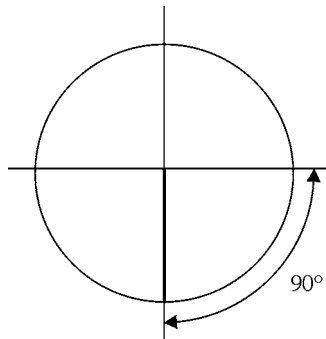
c)  $Z_3 = 10 \angle -30^\circ = 10 \cdot \cos(-30) + j10 \cdot \sin(-30) = 8,66 - j5$



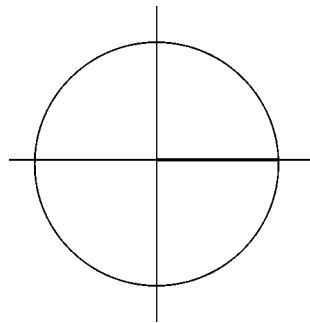
d)  $Z_4 = 25 \angle 90^\circ = 25 \cdot \cos 90 + j25 \cdot \sin 90 = 0 + j25 = j25$



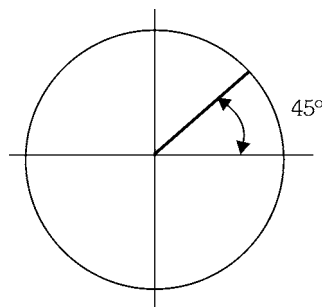
e)  $Z_5 = 45 \angle -90^\circ = 45 \cdot \cos(-90^\circ) + j45 \cdot \sin(-90^\circ) = 0 - j45 = -j45$



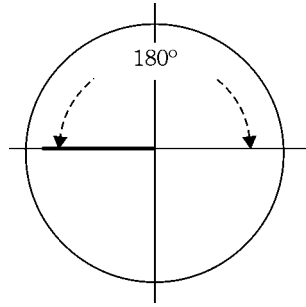
f)  $Z_6 = 220 \angle 0^\circ = 220 \cdot \cos 0^\circ + j220 \cdot \sin 0^\circ = 220$



g)  $Z_7 = 3,56 \angle 45^\circ = 3,56 \cdot \cos 45^\circ + j \cdot 3,56 \cdot \sin 45^\circ = 2,52 + j2,52$



$$h) 67 \angle 180^\circ = 67 \cdot \cos 180^\circ + j \cdot \sin 180^\circ = -67$$



### 1.3 - Operações com números complexos

$$Z_1 = 40 - j100 \quad Z_2 = 50 \angle 30^\circ \quad Z_3 = 5 + j8,66 \quad Z_4 = -20 - j40$$

$$Z_1 = 107,7 \angle -68^\circ, \quad Z_2 = 50 \cos 30^\circ + j50 \sin 30^\circ = 43,3 + j25, \quad Z_3 = 10 \angle 60^\circ$$

$$Z_4 = 44,7 \angle 243,4^\circ$$

Efetuar as operações:

$$a) Z_1 + Z_2 = (40 - j100) + (43,3 + j25) = 83,3 - j75$$

$$b) Z_1 + Z_4 = (40 - j100) + (-20 - j40) = 20 - j140$$

$$c) Z_2 + Z_4 = (43,3 + j25) + (-20 - j40) = 23,3 - j15$$

$$d) Z_1 - Z_2 = (40 - j100) - (43,3 + j25) = 3,3 - j125$$

$$e) Z_2 - Z_3 = (43,3 + j25) - (5 + j8,66) = 38,3 + j16,34$$

$$f) Z_3 - Z_4 = (5 + j8,66) - (-20 - j40) = 25 + j48,66$$

$$g) Z_3^2 = Z_3 \cdot Z_3 = 10 \angle 60^\circ \cdot 10 \angle 60^\circ = 100 \angle 120^\circ = 100 \cdot \cos 120^\circ + j \sin 120^\circ = -50 + j86,6$$

$$h) Z_1 \cdot Z_3 = 50 \angle 30^\circ \cdot 10 \angle 60^\circ = 500 \angle 90^\circ = j500$$

$$i) Z_4 / Z_1 = (44,7 \angle 243,4^\circ) / (107,7 \angle -68^\circ) = 0,415 \angle 213,4^\circ = 0,415 \cdot \cos 213,4^\circ + j0,415 \cdot \sin 213,4^\circ = -0,34 - j0,25$$

$$j) (Z_1 \cdot (Z_2 + Z_3)) / Z_4, \quad Z_2 + Z_3 = (43,3 + j25) + (5 + j8,66) = 48,3 + j33,66 = 58,8 \angle 34,8^\circ$$

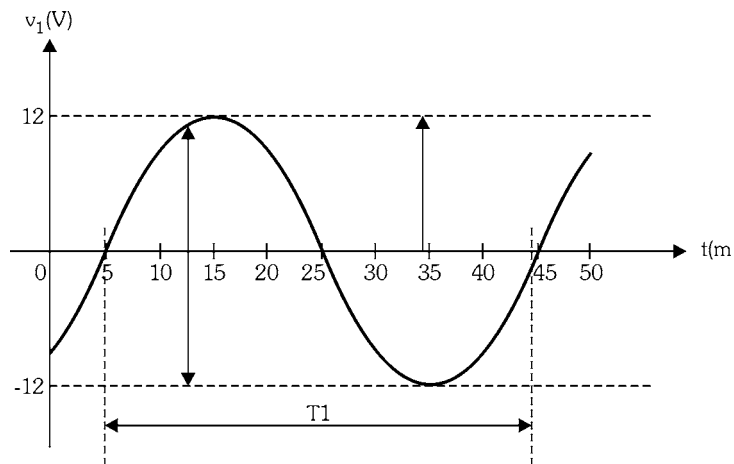
$$(107,7 \angle -68^\circ \cdot 58,8 \angle 34,8^\circ) / 44,7 \angle 243,4^\circ = 141,7 \angle -276,6^\circ =$$

$$141,7 \cdot \cos(-276,6^\circ) + j141,7 \cdot \sin(-276,6^\circ) = 16,3 + j140,7$$

## Capítulo 2

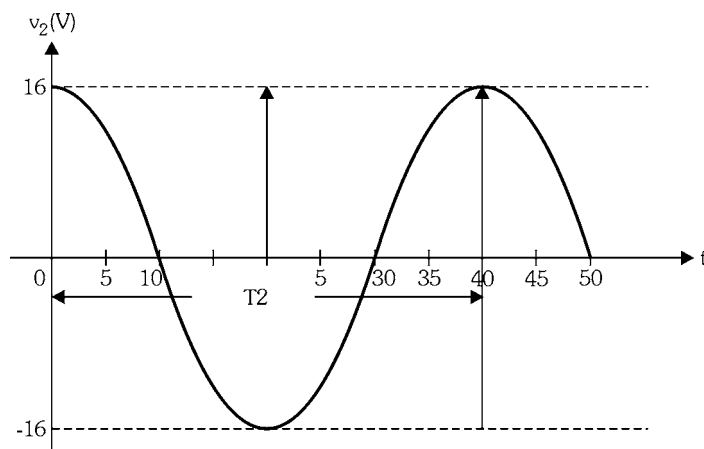
2.1 - 1 - Dadas as tensões representadas pelos gráficos seguintes, pede-se determinar:

- Valor de pico a pico
- Período, frequência e frequência angular
- Fase inicial e defasagem entre eles
- Expressão matemática



a) Dos gráficos obtemos:

Tensão1:  $V_{P1} = 12\text{V}$   $V_{PP1} = 24\text{V}$  tensão2 :  $V_{P2} = 16\text{V}$   $V_{PP2} = 32\text{V}$



b) Período

Tensão1: **40ms** (45ms -5ms)      tensão2: **40ms**

freqüência

tensão 1:  $f_1 = 1/40\text{ms} = 25\text{Hz}$      $f_2 = 1/40\text{ms} = 25\text{Hz}$

$\omega_1 = \omega_2 = 2 \cdot \pi \cdot 25 = 157 \text{ rd/s}$

c) Fase inicial

Tensão 1: para  $t = 5\text{ms} \Rightarrow v(5\text{ms}) = 0 = 12 \cdot \text{sen}(157 \cdot 5 \cdot 10^{-3} + \theta_{01}) \Rightarrow$

Significa que  $\text{sen}(0,785 + \theta_{01}) = 0 \Rightarrow 0,785 + \theta_{01} = 0 \Rightarrow \theta_{01} = -0,785\text{rd}$  ou  $\theta_{01} = -45^\circ$

Tensão 2: para  $t = 0 \Rightarrow v(0) = 16\text{V} = 16 \cdot \text{sen}(157 \cdot 0 + \theta_{02})$ , usando o mesmo raciocínio obtemos:  $\theta_{02} = 90^\circ$

Portanto, a defasagem entre elas é  $\Delta \theta_0 = 90 - (-45) = 135^\circ$

Tente desenhar o diagrama fasorial, representando as duas tensões.

d)  $v_1(t) = 12.\text{sen}(2.\pi.25.t - 45^\circ)(V) = 12.\text{sen}(50.\pi.t - 45^\circ)(V) = 12.\text{sen}(157.t - \pi/4)(V)$   
 $v_2(t) = 16.\text{sen}(50.\pi.t + 90^\circ)(V) = 16.\text{sen}(157.t + \pi/2)(V)$

2.2 - Tensão senoidal:  $f = 100\text{Hz}$   $V_p = 10$   $\theta_0 = -\pi/3 \text{ rd} = -60^\circ$

a)  $T = 1 / f = 1 / 100 = 0,01\text{s} = 10\text{ms}$   $\omega = 2.\pi.100 = 628 \text{ rd/s}$

b)  $V(t) = 10.\text{sen}(628.t - 60^\circ) (V) = 10.\text{sen}(628.t - \pi/3) (V)$

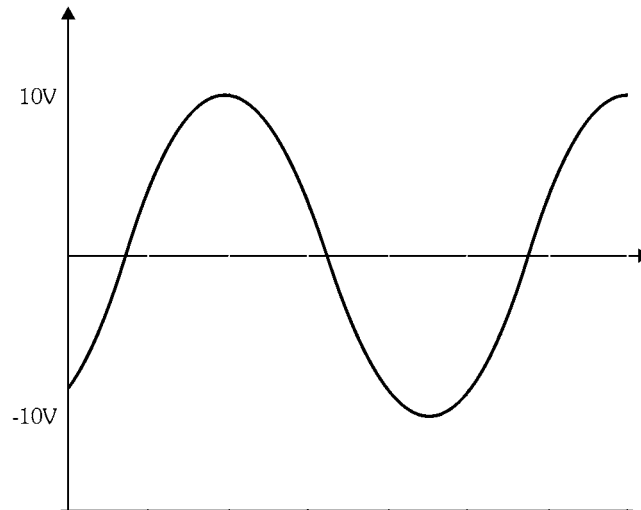
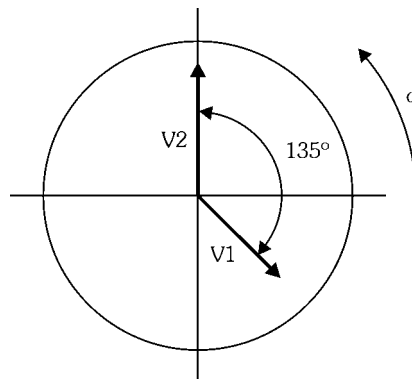


Diagrama Fasorial

2.3 -  $V_1 = 12 \angle -45^\circ$   $V_2 = 16 \angle 90^\circ$



Representação de tensão usando número complexo

2.4 -  $v_1(t) = 12.\text{sen}(50.\pi.t - 45^\circ)(V) = 12 \angle -45^\circ = 12.\cos(-45^\circ) + j.12.\text{sen}(-45^\circ) =$   
 $= 8,48 - j8,48 (V)$

$v_2(t) = 16.\text{sen}(50.\pi.t + 90^\circ)(V) = 16 \angle 90^\circ = j16 (V)$  não tem parte real !!

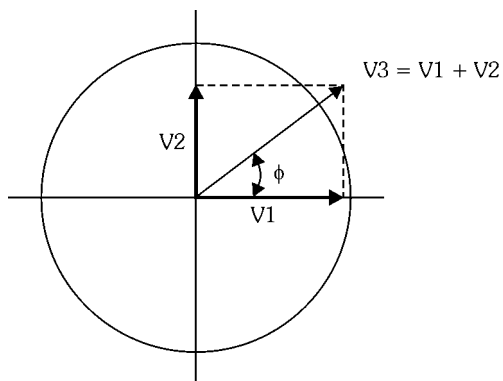
## Operações com Diagrama Fasorial e Números Complexos

$$2.5 - V1 = 30 \angle 0^\circ = 30 \text{ (V)}$$

$$V2 = 20 \cdot \text{sen}(w.t + \pi/2) \text{ (V)} = j20 \text{ (V)}$$

Obter:

a)  $V3 = V1 + V2$  fasorialmente



$$V3 = \sqrt{(20)^2 + (30)^2} = 36V \text{ com fase dada por: } \text{tg } \phi = V2 / V1 = 20/30 = 0,666 \text{ logo}$$

$$\phi = 33,7^\circ \Rightarrow V3 = 36 \angle 33,7^\circ \text{ (V)}$$

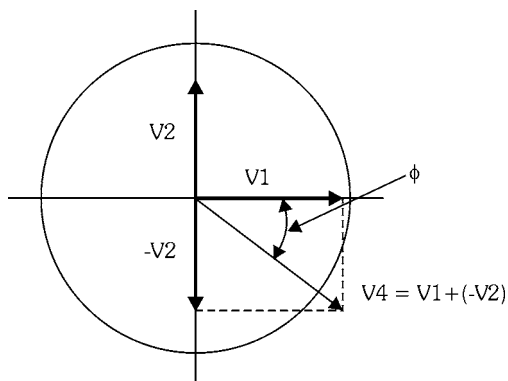
b)  $V3 = V1 + V2 = 30 + j20 \text{ (V)}$

c)  $V(t) = V1(t) + V2(t) = 30 \cdot \text{sen}(w.t) + 20 \cdot \text{sen}(w.t + 90^\circ) = 36 \cdot \text{sen}(w.t + 33,7^\circ) \text{ (V)}$

d) Na soma ponto a ponto, em cada instante somamos  $V1$  com  $V2$ , por exemplo no instante  $t = 0$   $V1(0) = 0$  e  $V2(0) = 20V$ , portanto  $V3(0) = 0 + 20 = 20V$  e assim por diante. Tente fazer com papel milimetrado.

e)  $V4 = V1 - V2$  fasorialmente

Observe que  $V1 - V2$  é a mesma coisa que  $V1 + (-V2)$ ,





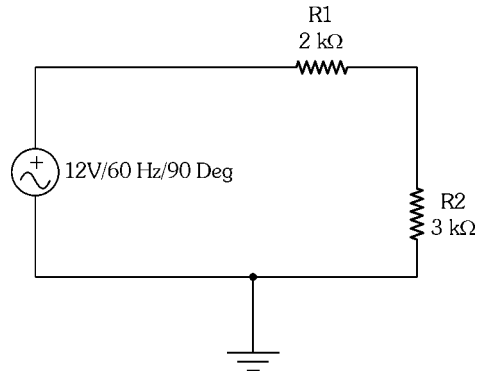
- f) Observe que o módulo de  $V_4$  é igual ao módulo de  $V_3$ , mas com fase diferente, logo podemos escrever as expressões de  $V_4$  na forma cartesiana e polar:

$$V_4 = 36 \angle -33,7^\circ \text{ (V)} \text{ e } V_4 = 36 \cdot \cos(-33,7^\circ) + j \cdot 36 \cdot \sin(-33,7^\circ) = 30 - j20 \text{ (V)}.$$

- g)  $V_4(t) = 36 \cdot \sin(\omega \cdot t - 33,7^\circ) \text{ (V)}$

Circuitos Resistivos em CA

2.6 - Dado o circuito, determinar:



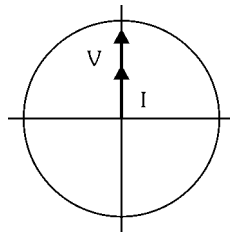
- a) Expressões de  $v(t)$  e  $i(t)$

A resistência equivalente é  $R_E = 5K$  e a corrente no circuito será dada por:

$$I = V/R_E = (12 \angle 90^\circ) / (5 \angle 0^\circ) = 2,4 \angle 90^\circ \text{ (mA)} = 2,4 \cdot \sin(\omega \cdot t + 90^\circ) \text{ (mA)}$$

$$V(t) = 12 \cdot \sin(\omega \cdot t + 90^\circ) \text{ (V)}$$

- b)



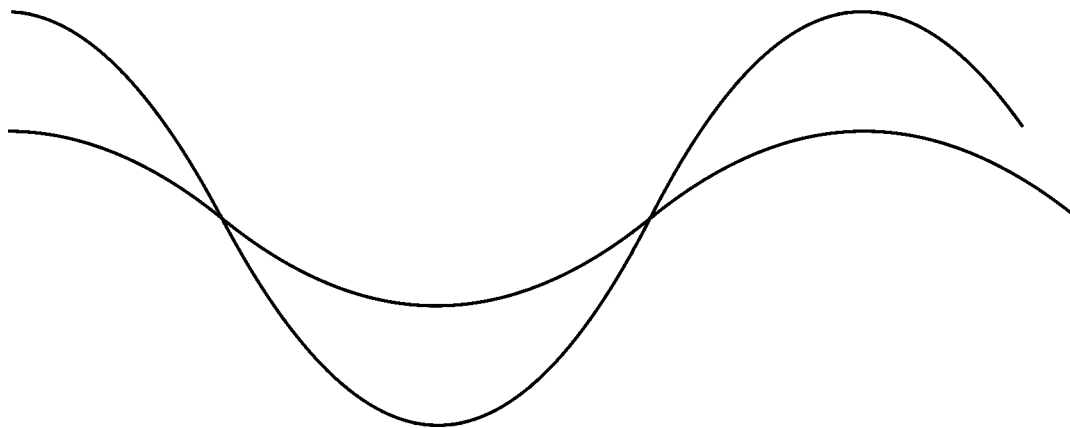
- c)  $V_1 = R_1 \cdot I = 20 \angle 0^\circ \cdot 2,4 \text{ mA} \angle 90^\circ = 4,8 \angle 90^\circ \text{ (V)}$

$$V_1(t) = 4,8 \cdot \sin(\omega \cdot t + 90^\circ) \text{ (V)}$$

$$V_2 = R_2 \cdot I = 3K \angle 0^\circ \cdot 2,4 \text{ mA} \angle 90^\circ = 7,2 \angle 90^\circ$$

$$V_2(t) = 7,2 \cdot \sin(\omega \cdot t + 90^\circ) \text{ (V)}$$

- d) As duas tensões estão em fase.



e) Potências de pico:

$$P_{PG} = V_P \cdot I_P = 12V \cdot 2,4mA = 28,8mW$$

$$P_{P1} = V_{P1} \cdot I_P = 4,8V \cdot 2,4mA = 11,52mW$$

$$P_{P2} = V_{P2} \cdot I_P = 7,2V \cdot 2,4mA = 17,28mW$$

Potências médias:

$$P_{MG} = P_{PG} / 2 = 14,4mW$$

$$P_{M1} = P_{P1} / 2 = 5,76mW$$

$$P_{M2} = P_{P2} / 2 = 8,64mW$$

f) Veja a figura 2.9, página 51